



## Chapter 18

### Representation of Problem Schemata\*

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Early “brute force” theories in problem solving (Newell & Simon, 1972) concentrated on developing powerful heuristics to reduce a problem’s search space. The more recent “knowledge-based” trend, however, recognizes the importance of the potential solver’s problem representation (e.g., Greeno, 1977; Hayes & Simon, 1974). In this respect, problem difficulty is not attributed to the size of the problem search space, but rather to the problem solver’s interpretation of the problem. Simply, people have difficulties in solving a problem because they have not formed an optimal internal representation of it.

The purpose of the present study is to examine what is included in a problem solver’s representation from the original problem statement and to determine whether Pathfinder networks are useful for capturing a “snap-shot” glimpse of this problem interpretation.

### Schemata and Interpreting Problems

The notion of a “schema” is at the core of this research. A schema is an abstract high-level knowledge structure containing “slots” that are filled by specific elements. The schema is said to be structured in that it shows the relationships among these component slots. The importance of this cognitive structure has been emphasized throughout the literature on expert-novice differences in problem solving (e.g., Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980; Schoenfeld & Hermann, 1982).

It is important to identify both the problem structures and the processes by which information from the problem statement is transformed into the problem schema (cf., Dellarosa, 1985). It is unlikely that a problem solver will duplicate a problem statement exactly in memory. Rather, the solver will probably delete some irrelevant information from the stated problem in addition to adding some pre-existing knowledge or inferences in interpreting the problem.

Simon and Hayes (1976) developed a computer program, *Understand*, that attempts to simulate one aspect of this representation-formulation process. *Understand* makes judgments on the importance and relevance of concepts in the problem statement and creates a problem representation from selected portions of the original problem text. Although the internal representation corresponds closely to the givens and the structure of the presented problem statement, not all aspects from the text are included in the resulting schema.

One question at hand concerns exactly what is taken from the problem text. In this respect, problem-solving research shares objectives with research in text comprehension. Problem statements, whether written or verbal, are necessarily a form of prose. Research on memory for prose has shown that when an appropriate schema is activated, subjects

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have a more unified knowledge base with which to understand text (Bransford & Johnson, 1972; Dooling & Lachman, 1971; Owens, Bower, & Black, 1979; Thorndyke, 1977).

It has been suggested that one particular aspect that serves as a schema frame is the particular goal or motive of a character within a text. Owens, Bower, and Black (1979) have shown that the reader will scan the text primarily for the goal-relevant facts. Other details receive less attention and are established only weakly in memory. Like readers of text, problem comprehenders also organize their schemata around their goal by extracting goal-oriented material from the text. Because their particular goal is problem solution, it is hypothesized that their schemata should be organized around a framework reflecting the steps necessary to achieve that goal.

One problem-solving model that includes text comprehension has been developed by Kintsch and Greeno (1985). These authors present a processing model that simulates the construction of cognitive representations for word arithmetic problems. Building on van Dijk and Kintsch's work (1983), the model contains two main components. The text base consists of a microstructure pertaining to the text's concepts and a macrostructure corresponding to the essential ideas these concepts convey. The second component, the problem model, contains representations of the relations and entities expressed in the text base. This problem model "reflects knowledge of the information needed to solve the problem" (p. 111). In forming the problem model, the reader will delete irrelevant concepts from the text base, in addition to adding inferences not included. This problem model becomes especially important for complex story problems where it is necessary to distinguish between the real problem and the presented cover story.

In the present research, one such complex story problem was studied. The reported experiment demonstrates that solvers with different problem solution goals not only abstract different portions from the problem text, but also organize this information in a manner unlike solvers with a separate solution goal. In addition, this research demonstrates that Pathfinder networks are useful for capturing these different problem schemata.

### Hypotheses and Rationale for Experiment

The particular problem under study in the present work (the "horse race problem") was one that could lend itself to the formation of dual representations. Specifically, the nature of the problem was such that two very different questions could be answered from the information presented in one cover story. Because the questions were quite different (one presented a mathematical speed/distance problem, the other, a more abstract puzzle based on sorting out incorrect inferences), it was predicted that the cognitive structures would reflect these differences.

The actual cover story used for both questions was stated as follows:

A stranger approaches two men sitting on the side of the road with their horses. One horse is a mustang, the other an appaloosa. The men discuss their long and unusual horse race. The first hour of the race, they traveled at a constant rate of six m.p.h. But since they had agreed that the owner of the horse that crossed the finish line first would lose, they slowed down. For the next half hour they traveled at half the original speed. Eventually, when they could go no slower, the horses halted and the men got off. The sun begins to set in the distance. The stranger remarks on the race. Then each of the two men jumps on one of the horses. The men speed the last half mile at a rate four times as fast as the original speed. The sun sets behind a hilltop as the stranger continues on his way. (Adapted from Poser #53 in Kaplan, 1963)

This same problem cover story yielded two different problem questions. The Stranger question was stated as follows: "What might the stranger have suggested, consistent with the men's original agreement, to make the men speed toward the finish line?" (answer: The stranger suggested the men switch horses). In comparison, the Equation question asked, "How many total miles did the men race?" (answer:  $6 + 1.5 + .5 = 8$  miles).

The present experiment employed three experimental conditions. Subjects in the two Problem conditions were required to produce a solution to one of the two questions. The Problem question was presented prior to the cover story in order to activate the appropriate schema in a top-down fashion during the initial reading of the problem story. The Stranger problem subjects ( $n = 6$ ) read the Stranger question, followed by the problem cover story, and then were required to produce a solution to the stated problem. The Equation problem subjects ( $n = 6$ ) read the Equation question, followed by the problem cover story, and then attempted to answer this stated problem. After solutions were attempted, both of these Problem groups were required to recall as much of the initial problem story as possible. The control or Read subjects ( $n = 6$ ), on the other hand, were only required to read the original cover story without either of the two problem questions. They were not required to solve a problem, but rather, were only required to recall as much of the original problem cover story as possible.

It was hypothesized that the resulting cognitive structures would depend upon the experimental condition. Specifically, Problem subjects should develop a closely knit schema of ideas central to their solution, with irrelevant concepts attached loosely to this core. Predictions for Read subjects are less simple to make. It might be the case that even without a guiding problem question they can recognize one of the inherent problem statements and their schemata would be organized accordingly. This possibility seems plausible, considering the active nature of text comprehenders. However, it might also be the case that no higher order framework will be activated for Read subjects. In this case, Read subjects will have stored only disconnected fragments in memory, with no efficient way to access all of them in free recall. Perhaps a little of each problem story will be reflected in the resulting schema.

### Characteristics of the Cover Story

In a separate study, 33 judges were presented with the problem text and were asked to answer both problem questions. Each problem was appropriately difficult for the group under study (9 out of 33 subjects found the correct solution to the Stranger problem; 12 out of 33 solved the Equation problem correctly).

After attempting both problem solutions, subjects then provided a rating for each of the 12 statements in the problem text. Specifically, these judges indicated whether a given sentence was related to understanding only the Stranger problem (S), only the Equation problem (E), Both problems (B), or Neither problem (N). The results of this analysis are shown in Table 1. Conveniently, the ratings resulted in three statements for each of the four categories, presented in an ungrouped fashion in the original text. These ratings imply that only half of the statements are relevant to the Equation problem [the three Equation only (six mph, half speed, half mile) plus three Both (stranger approaches, men discuss, horses halt) statements]. Similarly, only six statements are relevant to the Stranger problem [the three Stranger only (agreement, stranger remarks, men jump) plus three Both (stranger approaches, men discuss, horses halt) statements]. Consequently, exactly half of the statements are not relevant to the solving of the problem. For the Equation problem, the Stranger only (agreement, stranger remarks, men jump) and Neither (mustang, sun sets, stranger continues) statements do not apply. For the Stranger problem, the Equation only



(six mph, half speed, half mile) and Neither (mustang, sun sets, stranger continues) statements are not relevant.

Table 1. Classification and abbreviations of story sentences.

Classification and Abbreviation	Sentence from Story
B - stranger approaches	A stranger approaches two men sitting on the side of the road with their horses.
N - mustang	One horse is a mustang, the other an appaloosa.
B - men discuss	The men discuss their long and unusual race.
E - six mph	The first hour of the race, they traveled at a constant rate of six mph.
S - agreement	But since they had agreed that the owner of the horse that crossed the finish line first would lose, they slowed down.
E - half speed	For the next half hour they traveled at half the original speed.
B - horses halt	Eventually, when they could go no slower, the horses halt and the men got off.
N - sun sets	The sun begins to set in the distance.
S - stranger remarks	The stranger remarks on the race.
S - men jump	Then each of the two men jumps on one of the horses.
E - half mile	The men speed the last half mile at a rate four times the original speed.
N - stranger continues	The sun sets behind a hilltop as the stranger continues on his way.

### Recall Study

Subjects in the Stranger and Equation conditions read their respective problem questions followed by the problem story and then attempted a problem solution, showing as much work as possible on the paper provided. Read subjects were asked to read the problem story "until they were sure they fully understood it." Then, all subjects were asked to "recall (into a tape recorder) as much of the original problem story as possible." It was predicted that the recall performance would reflect the particular assigned experimental condition.

Specifically, subjects who had a schema for the Stranger problem should recall more stranger relevant concepts than equation concepts, because when they read the story they extracted information from the text by trying to fit information into their extant schema, which had slots only for stranger information. Slots should not be available for irrelevant information, so should not be stored in a form organized for easy recall. In contrast, subjects who had a prior schema for the Equation problem should recall more equation concepts than stranger concepts because only equation information is in an easily accessible form in memory. Finally, Read subjects who were lacking an organized prior schema should show no recall preference for concept type because they had no bias in memory.

Between group recall predictions also follow the same rationale. Stranger subjects should recall more stranger concepts than Equation and Read subjects, and Equation subjects should recall more equation concepts than Stranger and Read subjects. Based on this prediction, it also follows that Read subjects should recall fewer concepts overall.

Recall order was assessed by two judges who were blind to experimental condition. Coding of verbal protocols is a particularly difficult task, but based on the following set of coding rules, judges reached 100% agreement. Specifically, the main concepts from each of the above statements were listed. These main ideas were specified by the two judges as follows:

<b>stranger approaches</b>	stranger approaches (meets, sees, etc.) two men two men are sitting on the side of the road two men have two horses
<b>mustang</b>	two types of horses OR mustang OR appaloosa
<b>men discuss</b>	men discuss a race
<b>six mph</b>	one hour OR six mph
<b>agreement</b>	the men make an agreement (essentially that the winner loses)
<b>half speed</b>	next half hour OR half speed OR three mph
<b>horses halt</b>	men go slower and slower OR horses halt OR men jump off
<b>sun sets</b>	sun begins to set
<b>stranger remarks</b>	stranger remarks (comments, says something, etc.)
<b>men jump</b>	men jump back on the horses
<b>half mile</b>	four times as fast OR last half mile
<b>stranger continues</b>	sun sets OR stranger continues

Subjects were given credit for the particular statement whenever the gist of its main concept was recalled. In this manner, a given statement could be represented at various times throughout the protocol. The coding was fairly strict, however. According to this coding scheme, for example, in order for subjects to receive credit for recalling the fourth statement (six mph), they were required to mention the actual speed (six mph); they were not given credit if they vaguely mentioned just "some speed."

As the reader will note, this analysis is quite different from typical propositional analyses of semantic relations (e.g., Kintsch, 1974). Specifically, not every proposition receives credit. This limited propositional analysis was required for several reasons: to keep the number of main ideas relevant to both problems equal; to highlight the important aspects of the text; and to keep the number of concepts from becoming too large and unwieldy. For example, if a subject vaguely remembered that the horses were traveling some speed but could not remember the exact number, then it can be assumed that this subject is lacking important information from the problem statement relevant to producing the correct solution. If this subject was given the same credit as a subject who had actually recalled that the exact speed was six mph, then the transcription would not reflect these differences. Rather than assigning degree of accuracy ratings for each proposition, for simplicity, in the present analysis, it was decided that either the subject recalled the main point of the statement or did not recall it at all.



### Recall Analysis

The specific items recalled by each group were examined. Table 2 presents the mean number of statements recalled by the experimental group for the four statement categories.

Table 2. Mean number of items recalled in each concept category by each condition.

Instruction Condition	Concept Category				Average
	Stranger Related		Stranger Unrelated		
	Equation Related (Both)	Equation Unrelated (Stranger)	Equation Related (Equation)	Equation Unrelated (Neither)	
Stranger	2.06	2.50	1.75	1.25	1.89
Equation	1.37	1.69	2.69	1.69	1.86
Read	2.25	2.06	1.94	1.69	1.99
Average	1.89	2.08	2.13	1.54	

An analysis of variance was performed on a  $2 \times 2 \times 3$  two-within, one-between subjects design. The two-within subjects variables pertained to statement category: Stranger (Related or Unrelated) and Equation (Related or Unrelated). The between subject variable was experimental Condition (Stranger, Equation, or Read).

An ANOVA performed on the above design yielded only two significant effects. A significant interaction for the Stranger Category  $\times$  Condition was found ( $F(2,45) = 8.37$ ,  $p < .001$ ). This interaction is presented in Table 3.

Table 3. Mean number of items recalled by experimental condition for *Stranger Related* and *Stranger Unrelated* concepts.

Condition	Concept Category		Average
	Stranger Related (Both/Stranger)	Stranger Unrelated (Equation/Neither)	
Stranger	2.28	1.50	1.89
Equation	1.53	2.19	1.86
Read	2.16	1.82	1.99
Average	1.99	1.84	

Simple effects analyzed for Stranger Related items were significant ( $F(2,45) = 6.09$ ,  $p < .01$ ). A Newman-Keuls analysis indicated that although the Stranger condition did not differ from the Read condition, both groups recalled significantly more Stranger Related items than the Equation condition. No simple effect was found for the Stranger Unrelated items, although this comparison did approach significance ( $p < .07$ ), hinting that the Equation condition recalled more Stranger Unrelated concepts than the Read or Stranger groups.

In addition to the above interaction, the Stranger Category  $\times$  Equation Category interaction was also significant ( $F(1,45) = 10.08$ ,  $p < .01$ ). This interaction is presented in Table 4.

Table 4. Mean number of items recalled by concept category.<sup>a</sup>

	Equation Related	Equation Unrelated	Average
	(Both)	(Stranger)	
Stranger Related	1.89	2.08	1.99
Stranger Unrelated	(Equation) 2.13	(Neither) 1.54	1.83
Average	2.01	1.81	

<sup>a</sup>Category abbreviation in parentheses

Paired  $t$ -tests indicated significant differences for the Stranger Unrelated simple effect ( $t(47) = 2.96$ ,  $p < .01$ ). Specifically, Equation concepts were more likely to be recalled than Neither concepts. No simple effect was found for the Stranger Related items: Stranger Only concepts were just as likely to be recalled as Both concepts.

No main effects for Condition, Stranger category, or Equation category were found. In addition, although the Stranger  $\times$  Equation  $\times$  Condition interaction was insignificant, the Equation  $\times$  Condition interaction approached significance ( $p < .10$ ). This Equation  $\times$  Condition interaction showed that, although not statistically significant, the Equation subjects had a tendency to recall more Equation Related and less Equation Unrelated concepts than the Stranger and Read groups.

### Recall Conclusion and Discussion

As the above analysis indicates, essentially all groups are recalling the same number of items. Interestingly, there were significant differences in the specific concepts recalled: Although groups recalled about the same number of concepts, the particular concepts recalled depended on condition. Specifically, both the Stranger and Read conditions recalled significantly more Stranger related items than the Equation subjects, and Equation subjects tended to recall more Equation related items than the Stranger and Read subjects.

These findings met the predictions made by the literature on memory for prose. When given a specific theme for a passage, items relevant to that theme are more likely to be recalled. In problem solving, the particular "theme" is represented by the problem solution required of the potential solver. As the above recall analysis indicates, subjects do recall more concepts that are relevant to their problem goal—the concepts required to produce the solution. This explanation also helps us understand why the concepts labeled Neither were the least likeliest to be recalled, as these concepts are not required for either solution. At







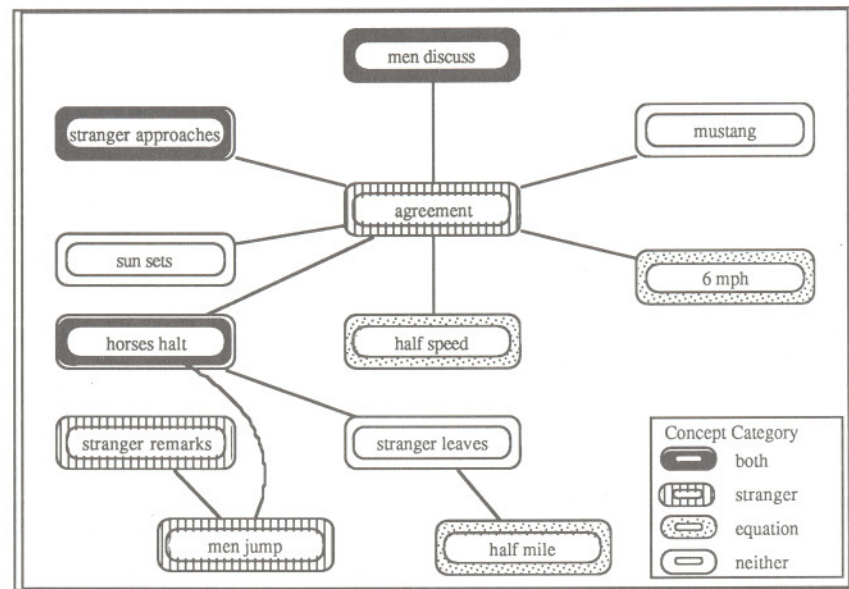


Figure 3. Pathfinder graph from Read Group average ratings.

An ANOVA performed on the original recall data supplements these findings. Specifically, one can pair each of the three Equation concepts with each other and examine the actual recall distances obtained for each of these three pairs across groups. An ANOVA indicated that the groups did differ in how far apart the two items in the pair were recalled from each other: (*six mph-half speed*): ( $F(2,45) = 4.44, p < .01$ ); (*six mph-half mile*): ( $F(2,45) = 10.33, p < .001$ ); (*half mile-half speed*): ( $F(2,45) = 7.61, p < .001$ ). Analytical comparisons indicated that Equation subjects had significantly shorter pairwise distances for all three combinations than did Read or Stranger subjects.

Similarly, an analysis of the three Stranger pairwise distances showed differences in recall performance, but only for two of the three pairs: (*agreement-stranger remarks*): ( $F(2,45) = 4.94, p < .01$ ); (*agreement-men jump*): ( $F(2,45) = 5.61, p < .01$ ). Analytical comparisons indicate that for the first pair, Stranger subjects recalled the two items closer in time than did the Equation and Read subjects. For the second pair, Stranger subjects only differed significantly from the Equation subjects.

Returning to Figures 1-3, as the graphs illustrate, the concepts labeled relevant to Both problems are attached to different nodes across the three graphs. Specifically, in the Equation graph, Both nodes are closer to Equation nodes than to any of the three other types of nodes. Similarly, in the Stranger and Read graphs, the Both nodes are more likely to be directly attached to Stranger nodes than to any other node type. This finding makes sense in terms of schema theory described above. Once text comprehenders have an idea about the meaning of the prose, it is possible to interpret incoming text around that idea. Ambiguous information, or information that is appropriate to two competing problems, in this case, migrates to other information relevant to the problem at hand.

The above qualitative description was confirmed quantitatively through ANOVA's performed on the original recall data. Each of the three concepts relevant to Both problems were first paired with each of the three Stranger relevant concepts. Of these nine combinations, for five of them, Stranger subjects had shorter recall distances between the pair than did Equation subjects, although only one shorter than Read subjects. Similarly, the three Both concepts were then paired with the three Equation concepts. Of these nine pairs, Equation subjects had significantly shorter recall distances between three of them than did Stranger subjects, although only one shorter than Read subjects.

The above findings illustrate that subjects within the two Problem groups are performing quite differently from each other. Specifically, subjects appear to be chunking problem relevant information together. These closely knit subschemata form the core of the problem representation. Subjects are hence quite capable of disentangling problem relevant information from irrelevant distractors in the cover story. Memory organization is quite efficient in that problem solvers need only pay attention to the core of the representation, and can successfully ignore the less central slots.

A final analysis examined the similarity of the graphs for the three groups (see Goldsmith & Davenport, Chapter 5, this volume). The intersection/union ratio which ranges from 0 (no commonality) to 1 (identical) was used. The average intersection/union ratios for the three comparisons were as follows: Stranger-Equation = .29; Stranger-Read = .57; Equation-Read = .35. An ANOVA indicated that these differences were significant ( $F(2,33) = 9.38, p < .001$ ). A paired *t*-test showed that the comparison Stranger-Equation versus Stranger-Read was significant ( $t(11) = 4.23, p < .01$ ), and that Stranger-Read versus Equation-Read was significant ( $t(11) = 2.94, p < .05$ ). These results indicated that there were differences in the way the three groups represented the relationships among the 12 concepts. Specifically, the Stranger-Read representations were significantly more alike than any of the other pairs.

## Conclusion and Discussion

In this research, problem representation was captured by empirically derived networks for three experimental groups. Although the two problem groups read the same problem cover story, each group read the story with a different goal in mind. The resulting PFNETs were organized around and reflected these separate goals. Specifically, goal relevant information was highly interwoven, forming the center of the networks with irrelevant information loosely attached to this schematic core.

Previous research on memory for prose has indicated that readers of text are more likely to recall thematically relevant portions of the text in comparison to unrelated aspects of the prose. It has been suggested that the use of a schema is responsible for such biases. Specifically, upon being given a theme for the passage, a high-level knowledge structure or schema becomes activated. This mechanism contains slots readily filled with thematically relevant concepts. Unrelated concepts have no slots in this overall schematic representation, and are thus stored in unconnected fragments in memory. Because they are not available in a highly accessible form, these unrelated concepts result in hindered recall performance. In contrast, those concepts that have been organized into a meaningful high-order schema are more readily available during recall. Although this schema explanation makes sense at an intuitive level, confirmation of its existence has been difficult due to an inability to directly examine these knowledge structures.



Just as the schema has been considered the main factor in determining recall for prose, it has also been suggested that this knowledge structure is the most important determinant of problem-solving performance. In fact, the knowledge-based analysis in problem solving recognizes the importance of the problem solver's representation. As in memory for prose, however, techniques for capturing one's problem representation have not been as direct as desired.

The present research has combined the above two areas of research by examining the extraction of problem relevant information from text. The obtained results support the notion of a schema-like mechanism for influencing problem perception and the way incoming information is being interpreted. For example, this research has shown that readers of problems are more likely to recall aspects of the problem story that are relevant to the overall theme. In problem solving, the goal state or the solution required serves as this schematic organizer. This research also demonstrates that Pathfinder is useful for capturing these knowledge representations.

Pathfinder networks demonstrated the major differences between problem solvers with different goals. Although the Stranger group and the Equation group both read the same cover story, this story resulted in different memory organizations for the two groups, as captured by the empirically derived networks. Specifically, the problem representation was organized around the specific goal or solution required of its potential solver. Oddly, few differences were found between the subjects who had the Stranger theme for the problem story and those who had no theme at all. The literature on memory for prose would predict that these latter Read subjects would have no overall schema activated with which to understand the incoming text. Thus, their recall performance should have been hindered. This prediction was not met in the current experiment, however. In fact, the resulting Read network was highly similar to the Stranger network, although differed considerably from the Equation network. One explanation for this finding is that even without being given a problem question, Read subjects could recognize an underlying problem statement without being cued for it directly. One reason why the Stranger problem might have jumped out at its reader is because this problem was so unusual and was actually the original problem to which the Equation problem was added. This explanation seems plausible, as many of the Read subjects actually asked the experimenter what the stranger suggested after the experiment was over. Further research on different problem cover stories would enhance our understanding of this result. It might be especially interesting to capture the resulting schemata for problem groups that read the same problem story made up of two story problems rather than one story problem and one mathematical problem.

Further research might also examine the ability to retrieve irrelevant concepts. For example, in this research, problem subjects were more likely to recall relevant concepts than irrelevant concepts. This experiment did not determine whether or not irrelevant information was completely unavailable, however. It might be the case, for example, that separate schema slots exist for unimportant concepts and these ideas are simply stored as such. It might be interesting to see if more accurate recall could be obtained by allowing some time to elapse and then prompting subjects with the problem question opposite to the problem they actually worked on. In this way, one could also obtain information that would help disentangle whether or not the problem question aided schema formation during encoding versus during retrieval (i.e., by providing an appropriate framework with which to access or retrieve the slots). Once again, in directly comparing the differences between the schema formed during encoding versus the schema formed during retrieval, Pathfinder networks would be useful.