

Dedicated to the memory of Professor Xiao-Song Lin



July 27, 1957—Jan. 14, 2007

FOREWORD

This volume through its diversity reflects Lin's journal-editing side. In addition to the mathematics he did he always read widely and with good taste in and around all the lovely terrain which neighbors on topology. Topology, in this way is a good subject for an eclectic. There is almost no domain from condensed matter physics to p-adic analysis that one cannot find a topological excuse to study. Lin, the journal editor, did this. I hope he would find this volume interesting and thoughtful enough that he might have put it together himself.

Lin was, with Steve Hickling, one of my first two graduate students. So he and I figured out the thesis advising process together. I needed a lot of help. I had had an unusual education and had missed some of the steps. To my great relief Lin seemed to know how it would go. He would talk to me about what I was working on, focus in on something that I was not understanding properly and then dig in. As with most great students the advisor has little to do with the thesis. Lin had a great idea, which he developed with Habegger, and is further expanded in this volume. The idea was very reductionist, he thought the subject of knots and links had begun in the wrong place, as if physicists had tried to solve for H_2^+ without doing the hydrogen atom first. His idea was to study "string links" : arcs in a ball with fixed boundary conditions, rather than ordinary links in S^3 . First thing first. Do the local problem before going global. It was a very sensible idea and amazing that knot theory had existed 100 years without this idea popping up. I think, 20 years later in 2008, this idea would be considered obvious. But this is a sign that Lin and a few like him exerted a systematic influence: localizing and systematizing geometric topology. If you like making topology more like quantum field theory.

Quantum field theory is another area where Lin was in the vanguard. The idea that some (but not all) classical link invariants are naturally described as coefficients of a perturbative expansion was sorted out between Bar-Natan and Lin in the early 1990s leading quickly to the theory of finite type invariants. It was a thrill for me to see how Lin took to the then new

subject of quantum topology. At first I did not expect to learn this subject myself. However in the mid 90s I was thinking about building a computer based on the Chern-Simons Lagrangian and Lin became my tutor, explaining the Jones representations, and all the related algebras. He made the subject very friendly. For over a decade he would stay in touch what he was doing and thinking in the “Jones” world. His work in this area is among the finest and this volume serves, also, as a tribute to these contributions.

Finally, it is not well known but Lin made a serious effort to understand Perelman’s proof of Thurston’s geometrization conjecture and the many ancillary expositions. The article herein on generalized Ricci flow commemorates his efforts both as a researcher and journal editor here.

Lin was a great and generous spirit, well loved by our community. He was our dear friend. We miss him and dedicate our work to him.

Michael H. Freedman

PREFACE

On January 14, 2007, our beloved friend Xiao-Song Lin left us. On July 27, 2007—his 50th birthday, his friends, colleagues and family members gathered in the Chern Institute of Mathematics to celebrate his wonderful life. These proceedings resulted from this conference, and is a permanent tribute to a humble person, a true mathematician, a great friend, and a devoted family man.

Topology and physics are central themes in Xiao-Song's professional career. A central player in quantum topology is knot theory. Knot invariants such as the celebrated Jones polynomial and finite type invariants were constantly on Xiao-Song's mind. As one of the leading quantum knot theorists in the world, Xiao-Song made fundamental contributions to the development and popularization of knot theory. With his untimely death, the knot world lost a great leader.

During the international conference from July 27 to July 31, 2007 at the Chern Institute of Mathematics, Xiao-Song's friends and colleagues covered a variety of topics in topology and physics. We are sure that Xiao-Song will smile in heaven when the topics dear to his heart continue to flourish.

We thank all the participants and speakers for making the conference a memorable one. Staff in the Chern Institute won the hearts of the participants for staying on top of everything. We thank all of them, especially Mrs. Hongqin Li.

Zhenghan Wang
Weiping Zhang

Short Biography of Lin

Xiao-Song Lin, a Professor of Mathematics at the University of California at Riverside, died on January 14, 2007 in Riverside, California, six months after being diagnosed with advanced stage liver cancer. He was 49.

Xiao-Song Lin was born in Songjiang, Shanghai, on July 27, 1957, and grew up in Suzhou, Jiangsu. In 1984, he received his M.S. in Mathematics from Beijing University under the direction of Professor Boju Jiang. That same year, he arrived in the United States to study at the University of California, San Diego under Professor Michael H. Freedman. After obtaining his Ph.D. in 1988, he began his career at Columbia University. In 1995, he joined the faculty at the University of California, Riverside, where he remained until the time of his death.

Xiao-Song Lin was a mathematician of exceptional ability and creativity. His areas of specialization were in low-dimensional topology and quantum topology. He was best known for his numerous contributions to knot theory. Throughout his entire career, Xiao-Song Lin maintained a passionate commitment to mathematical research and the mathematics community. He was co-founder and co-Editor-in-Chief of the research journal *Communications in Contemporary Mathematics*, and served on the editorial boards of several others. He advised five Ph.D. students, and he served as a mentor to many other graduate and post-doctoral students in topology. He will always be remembered by his students and his colleagues for his patience, his generosity, and his willingness to share mathematical ideas.

Xiao-Song Lin received many honors and awards, including the prestigious Sloan Fellowship (1992-1994); he was a member of the Institute for Advanced Study (Spring 1988 and 1993-1994); he was a Professor of Special Mathematics Lectures at Beijing University (1998-2000); and he was named Beijing University's Chang Jiang Scholar (2006-2008) by the Chinese Ministry of Education.

Despite his employment in the USA, Xiao-Song Lin was actively involved in the advancement of Chinese mathematics and kept in close contact with the topology research group at Beijing University. Beginning in

the early 1990s, he spent most of his summers in China, primarily giving lectures and teaching classes at his alma mater Beijing University. Together with Professor Boju Jiang and Professor Shicheng Wang, he helped to plan and organize the annual Chinese Low-Dimensional Topology Summer School, the 2002 ICM Satellite Conference in Geometric Topology, as well as many other mathematical meetings and conferences in China.

His untimely death is a great loss to the international topology community, and to all who knew him. In his honor and in his memory, the Xiao-Song Lin Award was established by Xiao-Song Lin's family: Each year, a cash prize of at least 1000 USD will be awarded to a senior undergraduate at Beijing University who has demonstrated truly exceptional scholarship in mathematics.

Mathematics of Lin

Xiao-Song's first major work was a joint paper with Prof. M. Freedman on the A-B slice problem. In 1981, Freedman solved the 4-dimensional topological Poincaré conjecture. Actually, he achieved a complete classification of all closed simply-connected topological 4-manifolds. Later, Freedman proved that his method, in principle, works for a large class of fundamental groups including all finitely generated abelian groups. But he conjectured that his method will not cover the cases of non-abelian free groups. The A-B slice problem is a program to prove Freedman's conjecture. This difficult problem is still open. Prof. V. Krushkal's paper in this book gives an up-to-date account of this problem. The experience that Xiao-Song gained from this project strongly influenced his career and future research. In this paper, link homotopy was introduced into the study of 4-dimensional topology. Freedman recently wrote on this work: "Sometimes when I think of our work on that problem, I feel like an old time mountaineer stormed off a high peak just short of the summit."

After the A-B slice problem, it was natural for Xiao-Song to study link homotopy. In a joint work with Prof. N. Habegger, he solved a problem of Prof. J. Milnor from 1950s on the classification of links up to homotopy, where the notion "string link" was invented.

These two beautiful papers were essentially written during his graduate school years. After obtaining his Ph.D, he went to work at Columbia University. There in a joint paper with Prof. J. Birman, he axiomatized Prof. V. Vassiliev's knot invariants combinatorically, then expanded the Jones polynomial of knots into Vassiliev or finite type invariants. The Birman-Lin condition was discovered, and a second revolution in quantum knot theory (after Prof. V. Jones' first revolution) was started.

Xiao-Song's personal favorite work was his paper *A knot invariant via representation spaces*. Following an idea of Prof. A. Casson, Xiao-Song defined a knot invariant, which turned out to be the knot signature. Recently, this work was generalized using symplectic Floer homology by Prof. W. Li.

The paper *Representations of knot groups and twisted Alexander polyno-*

mials shows something that we are familiar with and grateful: Xiao-Song's generosity in mathematics. He introduced the twisted Alexander polynomials in this paper, but never rushed to publish the paper for the sake of credit. Actually this paper would never have been published if not solicited by an editor.

His unfinished work *Zeros of Jones polynomials* had captured his attention for a long time. Since the beginning of quantum knot invariants, Jones realized that his knot polynomial is related to statistical mechanics. In physics, the zeros of partition functions encode deep information about the corresponding physical systems. In his unfinished manuscript, Xiao-Song asked: how can one tell whether or not a Laurent polynomial with integer coefficients is the Jones polynomial of a knot? Then he wrote: maybe this was the wrong question. If so, what would be the right question? He believed that the right question would lead to some beautiful mathematics. At the end of the manuscript, he suggested to look for statistical laws for the norms and phases of the zeros of the Jones polynomials.

There are 38 publications by Xiao-Song listed in Mathematical Reviews so far, and 8 more papers are posted on the arXiv. Another 5 unpublished papers are collected in this volume. In addition, there are 4 more unfinished works on: the zeros of Jones polynomials, an L^2 -approach to the volume conjecture, wood puzzle games, and finger loop braids, respectively, that can be found on Xiao-Song's webpage <http://math.ucr.edu/~x1/>.

We cannot do justice to Xiao-Song's mathematics in a few pages. As Prof. D. Bar-Natan's talk title in Nankai said: following Lin. Then we are bound to discover beautiful gems in mathematics.

Quoting Freedman again: "There will ever after be string links and finite type invariants." We add: there will ever after be the name of Xiao-Song Lin in mathematics.

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